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Conformal Higgs model: charged gauge fields can produce a 125GeV resonance

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In the conformal Higgs model, the dynamical value of parameter λ in Lagrangian term $-\lambda(\Phi^\dagger\Phi)^2$ depends on the mass of an intermediate neutral boson field that combines interacting scalars $W_\mu^+ W_-^\mu$ and $Z_\mu^* Z^\mu$. If this mass is $125GeV$, λ is negative and of order 10^{-88}, in agreement with the empirical value deduced from well-established cosmological and electroweak data. Hence this intermediate scalar boson field is a candidate to explain the recently observed LHC resonance. The conformal Higgs model considers coupled fields: metric tensor $g_{\mu\nu}$, Higgs scalar Φ, neutral gauge boson Z_μ, and charged gauge bosons $W_\mu^\pm$. An earlier derivation, restricted to neutral Z_μ, is extended here to include charged fields $W_\mu^\pm$. λ is computed in a semiclassical approximation for fields coupled by the cosmological time-dependence of the gravitational Ricci scalar. The model predicts that the $125GeV$ resonance is accompanied by a broad resonance at $217GeV$.

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I. INTRODUCTION

The conventional Higgs model of electroweak physics postulates a scalar field Φ[1, 2] whose classical field equation has an exact stable solution of finite constant amplitude. Gauge symmetry defines covariant derivatives of Φ that couple it to neutral Z_μ and charged $W_\mu^\pm$ gauge boson fields. Gauge boson masses are determined at a semiclassical level by solving coupled field equations for the scalar and gauge boson fields[2].

Higgs Φ, of spacetime constant magnitude, is inherently a cosmological entity. Under a postulate of universal conformal symmetry for all bare elementary fields[3], nonzero scalar field Φ determines the nonclassical gravitational field responsible for Hubble expansion of a uniform, isotropic cosmos[4, 5]. The resulting conformal Higgs model[3, 6], limited to the neutral Z_μ field for simplicity, has been shown to be consistent with observed Hubble expansion and cosmic microwave background (CMB) parameters. Conformal gravity[4], recently applied to fit anomalous galactic rotation data for 138 galaxies[7–9], together with the conformal Higgs model and a conformal model of galactic halos[10], strongly suggest that for an isolated galaxy dark matter can be eliminated and dark energy explained by consistent conformal theory[4, 5].

Gauge boson masses depend only on the existence of a nonzero constant solution of the scalar field equation. This does not require a stable fluctuation of the scalar field, the usual definition of the long-sought Higgs boson[2]. The conformal Higgs model determines dark energy of the correct empirical magnitude[6], but does not imply a stable scalar field fluctuation, hence precluding a conventional massive Higgs boson[11]. This supports the conjecture that the recently observed LHC 125GeV resonance[12] is a hitherto unknown new particle or field. It will be shown here that extension of the conformal Higgs model to include charged gauge fields implies the existence of such an entity, consistent with a 125GeV resonance.

The conventional Higgs model postulates incremental Lagrangian density $\Delta\mathcal{L}_\Phi = w^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2$ for the scalar field[2]. The confomal Higgs model, restricted to gauge field Z_μ, has been shown to determine parameter w^2[6], which becomes a cosmological constant in the modified Friedmann gravitational field equation[3]. Extended here to include charged gauge fields $W_\mu^\pm$, conformal theory determines both parameters w^2 and λ. Estimated values are consistent with directly measured data from cosmology and electroweak particle physics. The extended theory implies existence of a candidate neutral field (or particle) that might account for the recently observed 125 GeV resonance[12], with properties anticipated for a massive Higgs boson.

Variational theory for fields in general relativity is a straightforward generalization of classical field theory[13]. Given scalar Lagrangian density $\mathcal{L}$, action integral $I = \int d^4x\sqrt{-g}\mathcal{L}$ is required to be stationary for all differentiable field variations, subject to appropriate boundary conditions. The determinant of metric tensor $g_{\mu\nu}$ is denoted here by g. Riemannian metric covariant derivatives $\mathcal{D}_\lambda$ are defined such that $\mathcal{D}_\lambda g_{\mu\nu} = 0$[4].

Conformal symmetry is defined by invariance of action integral $I = \int d^4x\sqrt{-g}\mathcal{L}$ under local Weyl scaling, such that $g_{\mu\nu}(x) \to g_{\mu\nu}(x)\Omega^2(x)$[14] for arbitrary real differentiable $\Omega(x)$, with fixed coordinates x^μ. For any Riemannian tensor, $T(x) \to \Omega^d(x)T(x) + \mathcal{R}(x)$ defines weight $d[T]$ and residue $\mathcal{R}[T]$. $d[\Phi] = -1$ for a scalar field. Conformal Lagrangian density $\mathcal{L}$ must have weight $d[\mathcal{L}] = -4$ and residue $\mathcal{R}[\mathcal{L}] = 0$, up to a 4-divergence[4].

Gravitational field equations are determined by metric functional derivative $X^{\mu\nu} = \frac{1}{\sqrt{-g}}\frac{\delta I}{\delta g_{\mu\nu}}$. Any scalar $\mathcal{L}_a$ determines energy-momentum tensor $\Theta_a^{\mu\nu} = -2X_a^{\mu\nu}$, evaluated for a solution of the coupled field equations.

Generalized Einstein equation $\sum_a X_a^{\mu\nu} = 0$ is expressed as $X_g^{\mu\nu} = \frac{1}{2}\sum_{a\neq g}\Theta_a^{\mu\nu}$. Hence summed trace $\sum_a g_{\mu\nu}X_a^{\mu\nu}$ vanishes for exact field solutions. Trace $g_{\mu\nu}X_a^{\mu\nu} = 0$ for a bare conformal field[4]. If interacting fields break conformal symmetry, vanishing of this summed trace is a consistency condition for solution of the field equations.

II. CONFORMAL SCALAR FIELD

The fundamental postulate that all primitive fields have conformal Weyl scaling symmetry is satisfied by spinor and gauge fields[15], but not by the conventional scalar field of the Higgs model[2]. A conformally invariant action integral is defined for complex scalar field Φ by Lagrangian density[3, 4]

$$\mathcal{L}_\Phi = (\partial_\mu\Phi)^\dagger\partial^\mu\Phi - \frac{1}{6}R\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2, \quad (1)$$

where R is the gravitational Ricci scalar. The conventional Higgs model[2] postulates incremental Lagrangian density $\Delta\mathcal{L}_\Phi$, which adds term $w^2\Phi^\dagger\Phi$ to $\mathcal{L}_\Phi$. Because this w^2 term breaks conformal symmetry, universal conformal symmetry requires it to be produced dynamically. The scalar field equation including $\Delta\mathcal{L}$ is[3] $\partial_\mu\partial^\mu\Phi = (-\frac{1}{6}R + w^2 - 2\lambda\Phi^\dagger\Phi)\Phi$. Neglecting second time derivatives of Φ, $\Phi^\dagger\Phi = \phi_0^2 = (w^2 - \frac{1}{6}R)/2\lambda$ generalizes the Higgs construction if this ratio is positive.

Because Ricci scalar R varies on a cosmological time scale, it induces an extremely weak time dependence of ϕ_0^2, which in turn produces source current densities for the gauge fields. The resulting coupled semiclassical field equations[6] determine nonvanishing but extremely small parameter w^2, in agreement with the cosmological constant deduced from observational data. This argument depends only on squared magnitudes of quantum field amplitudes. Although it breaks conformal symmetry, dynamically induced w^2 has been shown[6] to preserve the consistency condition that the trace of the total energy-momentum tensor should vanish[4]. It is shown here that biquadratic term $\lambda(\Phi^\dagger\Phi)^2$ is determined in conformal theory by charged gauge fields $W_\mu^\pm$.

III. CONFORMAL HIGGS MODEL RESTRICTED TO NEUTRAL GAUGE FIELD

The Higgs model[2] derives gauge boson mass from coupling via gauge covariant derivatives to a postulated SU(2) doublet scalar field Φ. SU(2) symmetry is broken by a solution of the scalar field equation such that $\Phi^\dagger\Phi = \phi_0^2$, a spacetime constant. Only the charge-neutral component of doublet field Φ is nonzero.

The essential result of the Higgs model, generation of gauge field masses, follows from a simplified semiclassical theory of the coupled scalar and gauge fields[2], extended here to include gravitational metric tensor $g_{\mu\nu}$ for the gravitational field. Nonlinear coupled field equations, greatly simplified by assuming the uniform, isotropic cosmological model of Robertson-Walker geometry[4, 5], are solved to obtain definite numerical results[3, 6]. For clarity, application of the conformal Higgs model restricted to neutral gauge field Z_μ is first summarized here, then the argument is extended to charged gauge fields $W_\mu^\pm$.

Gauge invariance replaces bare derivative ∂_μ by gauge covariant derivative $D_\mu = \partial_\mu - \frac{i}{2}g_z Z_\mu$[2], defining $g_z^2 = g_b^2 + g_w^2$. This retains $\mathcal{L}_Z$ and augments conformal $\mathcal{L}_\Phi^0 = (\partial_\mu\Phi)^\dagger\partial^\mu\Phi - \frac{1}{6}R\Phi^\dagger\Phi$ by coupling term $\Delta\mathcal{L} = (D_\mu\Phi)^\dagger D^\mu\Phi - (\partial_\mu\Phi)^\dagger\partial^\mu\Phi = \frac{1}{4}g_z^2\Phi^\dagger Z_\mu^* Z^\mu\Phi + \frac{i}{2}g_z Z_\mu^*\Phi^\dagger\partial^\mu\Phi - \frac{i}{2}g_z Z^\mu(\partial_\mu\Phi)^\dagger\Phi$. Weyl scaling residues in $\Delta\mathcal{L}$ cancel exactly for real gauge fields[16], so that the total energy-momentum tensor is conformal and traceless. The trace condition can be verified for the pure imaginary gauge field Z_μ derived here[6].

Derivatives due to cosmological time dependence act as an extremely weak perturbation of the Higgs scalar field. The scalar field is dressed by an induced gauge field amplitude. Derivatives of the induced gauge field (but not of Φ) can be neglected and are omitted from the derivations here.

For a massive vector field[2],

$$\partial_\nu Z^{\mu\nu} = 2\frac{1}{\sqrt{-g}}\frac{\delta\Delta I}{\delta Z_\mu^*} = m_Z^2 Z^\mu - J_Z^\mu. \quad (2)$$

$\Delta\mathcal{L}$ from covariant D_μ determines the field parameters:

$$2\frac{1}{\sqrt{-g}}\frac{\delta\Delta I}{\delta Z_\mu^*} = \frac{1}{2}g_z^2\Phi^\dagger\Phi Z^\mu + ig_z\Phi^\dagger\partial^\mu\Phi \quad (3)$$

implies $m_Z^2 = \frac{1}{2}g_z^2\phi_0^2$, verifying the Higgs mass formula. Defining real parameter $\frac{\dot\phi_0}{\phi_0}$, the fields are coupled by pure imaginary source density $J_Z^0 = -ig_z\phi_0^*\partial^0\phi_0 = -ig_z\frac{\dot\phi_0}{\phi_0}\phi_0^*\phi_0$. Neglecting derivatives of induced field Z^μ, the gauge field equation reduces to $m_Z^2 Z^\mu = J_Z^\mu$.

If $\Delta\mathcal{L}$ were equivalent to parametrized Higgs model $(w^2 - \lambda\Phi^\dagger\Phi)\Phi^\dagger\Phi$, the scalar field equation would be

$$\partial_\mu\partial^\mu\Phi + \frac{1}{6}R\Phi = \frac{1}{\sqrt{-g}}\frac{\delta\Delta I}{\delta\Phi^\dagger} = (w^2 - 2\lambda\Phi^\dagger\Phi)\Phi. \quad (4)$$

$\Delta\mathcal{L}$ from D_μ determines

$$\frac{1}{\sqrt{-g}}\frac{\delta\Delta I}{\delta\Phi^\dagger} = \frac{1}{4}g_z^2 Z_\mu^* Z^\mu\Phi + \frac{i}{2}g_z(Z_\mu^* + Z_\mu)\partial^\mu\Phi, \quad (5)$$

which implies $w^2 = \frac{1}{4}g_z^2 Z_\mu^* Z^\mu$. Pure imaginary Z_μ does not affect λ.

The standard Higgs model omits R and assumes $w^2, \lambda > 0$. $\Phi^\dagger\Phi = \phi_0^2 = w^2/2\lambda$ is an exact solution of the scalar field equation for any constant phase factor. This implies $\frac{\dot\phi_0}{\phi_0} = 0$, which does not couple the fields.

In conformal theory, solving the modified Friedmann equation determines Ricci scalar $R(t)$, which varies in cosmological time, but retains $\frac{1}{6}R - w^2 > 0$[3]. Hence,

for $\lambda < 0$, $\Phi^\dagger\Phi = \phi_0^2 = (\frac{1}{6}R - w^2)/(-2\lambda)$. The coupled nonlinear equations of conformal theory preclude a runaway solution for ϕ_0^2 regardless of the sign of λ[6]. Even if w^2 and λ remain constant, time-dependent R implies $\frac{\dot\phi_0}{\phi_0} = \frac{1}{2}\frac{\dot R}{R-6w^2} \neq 0$. From the modified Friedmann equation, fitted to redshift and CMB data and integrated to present time t_0[3, 6], $\frac{\dot\phi_0}{\phi_0}(t_0) = -.6053 \times 10^{-17}s^{-1}$. This implies small but nonvanishing source current J^μ_Z. From the scalar field equation, $w^2 = \frac{1}{4}g_z^2|Z|^2 = (\frac{\dot\phi_0}{\phi_0})^2$, in units such that $\hbar = c = 1$.

If charged gauge fields $W^\pm_\mu$ are omitted, parameter λ is not determined. w^2 and ϕ_0 are well-defined if $\lambda < 0$, as required by empirical $R > 6w^2$[6]. Confirming the standard Higgs model, gauge field Z_μ acquires mass m_Z from coupling to Φ.

The coupled fields break conformal and SU(2) symmetries. Time-dependent R implies nonvanishing real $\dot\phi_0$ and pure imaginary J^0_Z. Complex solutions of the gauge field equations, induced by pure imaginary current densities, exist but do not preserve gauge symmetry. Imaginary gauge field amplitudes model quantum creation and annihilation operators. Only the squared magnitudes of these nonclassical entities have classical analogs.

IV. CHARGED GAUGE FIELDS

In standard Higgs theory, it is assumed that charged component Φ_1 of the postulated SU(2) doublet scalar field vanishes identically, while $\Phi = \Phi_0 = \phi_0$ is a space-time constant. In conformal theory, ϕ_0 varies on a cosmological time scale[6]. Derived from the covariant derivatives of Φ[2], with $\Phi_1 \equiv 0$, the incremental Lagrangian density reduces to

$$\begin{aligned}\Delta\mathcal{L} &= (-\frac{ig_w}{\sqrt{2}}W_{\mu-}\Phi^\dagger)(\frac{ig_w}{\sqrt{2}}W^\mu_+\Phi)\\ &+(\partial_\mu + \frac{ig_z}{2}Z^*_\mu)\Phi^\dagger(\partial^\mu - \frac{ig_z}{2}Z^\mu)\Phi - \partial_\mu\Phi^\dagger\partial^\mu\Phi\\ &= (\frac{1}{2}g_w^2 W_{\mu-}W^\mu_+ + \frac{1}{4}g_z^2 Z^*_\mu Z^\mu)\Phi^\dagger\Phi\\ &-\partial_\mu\Phi^\dagger(\frac{1}{2}ig_z Z_\mu\Phi) + (\frac{1}{2}ig_z Z^*_\mu\Phi^\dagger)\partial^\mu\Phi. \qquad (6)\end{aligned}$$

Following the derivation given above for the neutral field only, this verifies the standard gauge mass formulae $m_Z^2 = \frac{1}{2}g_z^2\phi_0^2, m_W^2 = \frac{1}{2}g_w^2\phi_0^2$. $\Delta\mathcal{L}$ implies source current density $J^0_Z = -ig_z\frac{\dot\phi_0}{\phi_0}\Phi^\dagger\Phi$, which determines Higgs parameter $w^2 = \frac{1}{4}g_z^2 Z^*_\mu Z^\mu$. $W_{\mu-}W^\mu_+$ is shown below to depend explicitly on $\Phi^\dagger\Phi$, thus contributing to $-\lambda(\Phi^\dagger\Phi)^2$.

Although an independent field $W^\mu_\pm$ would violate charge neutrality, there is no contradiction in treating neutral two-component scalar field $WW = g_{\mu\nu}W^\mu_- W^\nu_+$ as an independent field or particle, in analogy to atoms, molecules, and nuclei. However, WW must interact strongly with corresponding neutral scalar field $ZZ = g_{\mu\nu}Z^{\mu*}Z^\nu$, through exchange of quarks and leptons. Assuming that the interacting bare fields produce relatively stable $W_2 = WW\cos\theta_x + ZZ\sin\theta_x$ and complementary resonance $Z_2 = -WW\sin\theta_x + ZZ\cos\theta_x$, W_2 can dress the bare Φ field while maintaining charge neutrality. This will be shown here to determine Higgs parameter λ.

$\Delta\mathcal{L}$ does not determine a direct source current density for bare field WW. However, simultaneous creation of paired fields Z^*_μ, Z^μ can produce the dressed field W_2. The rate of simultaneous excitation is the product of independent transition rates, expressed by $J_{ZZ}/\Phi^\dagger\Phi = (J^*_{Z\mu}/\Phi^\dagger\Phi)(J^\mu_Z/\Phi^\dagger\Phi)$. This implies $J_{ZZ}\Phi^\dagger\Phi = J^*_{Z\mu}J^\mu_Z = g_z^2(\frac{\dot\phi_0}{\phi_0})^2(\Phi^\dagger\Phi)^2$. Assuming a W_2 particle or resonance that contains the linear combination $WW\cos\theta_x + ZZ\sin\theta_x$, the effective source current is $J_{W_2} = J_{ZZ}\sin\theta_x$. Neglecting derivatives in an effective Klein-Gordon equation, the induced field amplitude is $W_2 = J_{W_2}/m^2_{W_2}$. Given $WW = W_2\cos\theta_x - Z_2\sin\theta_x$, W_2 projects onto WW with factor $\cos\theta_x$. Term $\frac{1}{2}g_w^2 WW\Phi^\dagger\Phi$ in $\Delta\mathcal{L}$ becomes $\frac{1}{4}g_w^2\sin 2\theta_x J_{ZZ}\Phi^\dagger\Phi/m^2_{W_2} = -\lambda(\Phi^\dagger\Phi)^2$, where $\lambda = -\frac{1}{4}g_w^2 g_z^2\sin 2\theta_x(\frac{\dot\phi_0}{\phi_0})^2/m^2_{W_2}$. Mass m_{W_2} must be consistent with empirical $\lambda \sim -10^{-88}$[6].

V. W_2 PARTICLE AND Z_2 RESONANCE

A model Hamiltonian matrix can be defined in which indices 0,1 refer respectively to bare neutral scalar states $WW = g_{\mu\nu}W^\mu_- W^\nu_+$, $ZZ = g_{\mu\nu}Z^{\mu*}Z^\nu$. The 2×2 submatrix with indices $0,1$ is assumed to be diagonal, with elements $H_{00} = 2m_W = 160GeV$, $H_{11} = 2m_Z = 182GeV$, for empirical masses m_W and m_Z. Neglecting non-Hermitian terms due to decay widths, intermediate quark and lepton states can be considered to define a submatrix $\tilde H$ of large dimension, indexed by $i,j \neq 0,1$, diagonalized with eigenvalues ϵ_i, and two vectors of off-diagonal elements $\tilde A_{i0}, \tilde A_{i1}$. $\tilde H$ projects onto energy-dependent off-diagonal elements of the 2×2 reduced matrix $H_{01}(\epsilon) = -\sum_{i\neq 0,1}\tilde A^\dagger_{0i}(\epsilon_i - \epsilon)^{-1}\tilde A_{i1}$, $H_{10}(\epsilon) = -\sum_{i\neq 0,1}\tilde A^\dagger_{1i}(\epsilon_i - \epsilon)^{-1}\tilde A_{i0}$. The reduced secular equation is $(H_{00}-\epsilon)(H_{11}-\epsilon) = H_{01}(\epsilon)H_{10}(\epsilon)$.

Taking some mean value for ϵ in elements H_{01}, H_{10}, they can be treated as empirical parameters, approximating the secular equation by $(H_{00}-\epsilon)(H_{11}-\epsilon) = |H_{01}|^2$. Eigenvector $(\cos\theta_x, \sin\theta_x)$ corresponds to eigenvalue $E_0 = H_{00} + H_{01}\tan\theta_x$ and eigenvector $(-\sin\theta_x, \cos\theta_x)$ corresponds to eigenvalue $E_1 = H_{11} - H_{10}\tan\theta_x$. Thus if $m_{W_2} = E_0 = 125GeV$, $H_{01} = H_{10} = -44.7GeV$,$\tan\theta_x = 0.784$, and $m_{Z_2} = E_1 = 217GeV$. Given an observed 125GeV resonance, the present analysis predicts a presumably broad resonance at 217GeV. The implied value of Higgs parameter λ is -0.50×10^{-88}, consistent with the empirical value inferred from cosmological data.

VI. CONCLUSIONS AND IMPLICATIONS

The postulate of universal conformal symmetry modifies both standard general relativity and the standard Higgs scalar field model. All mass terms and the analogous Higgs parameter w^2 break conformal symmetry and must be generated dynamically. The original Higgs model remains valid for gauge boson masses, but the negative sign of Higgs parameter λ implied by established cosmological data precludes a massive Higgs particle as a stable fluctuation of the conformal Higgs scalar field.

The coupled semiclassical field equations of conformal theory imply a very small but nonvanishing source density for the neutral gauge field Z_μ[5, 6]. This results from the cosmological time dependence of gravitational Ricci scalar R in the conformal Higgs scalar field Lagrangian density. Higgs scalar field Φ is dressed by a nonvanishing neutral gauge field, producing parameter w^2 of the correct magnitude for dark energy density as deduced from observed Hubble expansion [6].

The present paper extends this agreement with empirical data to Higgs parameter λ. Preserving charge neutrality, a virtual double excitation of the Φ field induces a previously unknown field W_2, based on strongly interacting bare fields $WW = W_{\mu-}W_+^\mu$ and $ZZ = Z_\mu^* Z^\mu$, that dresses the bare scalar field Φ, producing parameter λ of the correct empirical sign and magnitude if scalar field W_2 can be identified with the recently observed 125GeV LHC resonance.

By implication, the anticipated massive Higgs boson, absent from conformal theory, remains unobserved, and probably excluded by the prolonged experimental search[12]. If Higgs parameter w^2 were large enough to produce the observed 125GeV resonance, the conformal model would produce dark energy density so large as to blow the universe apart long before the present epoch. This paradox is removed by the present theory. The essential new feature is the existence of extremely small scale parameter $\frac{\dot\phi_0}{\phi_0}$, unique to conformal theory, which relates cosmology to electroweak physics.

Given a theory for Higgs parameter λ, it now becomes possible to carry solution of the modified Friedmann cosmic evolution equation back to the big-bang epoch, with time-dependent parameters. The reversed sign of the conformal gravitational constant in uniform, isotropic geometry[4, 5] implies rapid expansion due to the primordial mass and radiation density. The implied time variation of conformal Higgs amplitude ϕ_0 is relevant to nucleosynthesis, because it directly affects the Fermi constant for β-decay.

Dynamical models of galactic clusters should be revised to take into account the non-Newtonian long range gravitational effects of conformal halo theory[10]. It cannot yet be concluded that dark matter is needed to explain galactic evolution and cluster dynamics.

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